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Estimates of the Slovak zero-coupon yield curve

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Abstract

This note describes the procedure for estimating the term structure of interest rates. It documents the zero-coupon yield curve estimated from daily data for Slovak Government securities starting from January 2003. The estimation uses the methodology introduced in Nelson and Siegel (1987), and extended in Svensson (1994). We provide an aggregate measure of illiquidity derived from the pricing errors of coupon bonds.

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1 Introduction

This note describes the methodology for estimating the term structure of interest rates of Slovak Government debt securities. The term structure of interest rates is defined as a relationship between the maturity and the interest rate of zero-coupon bonds. Yield curves reflect the market expectations about the future evolution of short-term interest rates and the corresponding risk compensation. Obtaining and regularly publishing the zero-coupon curve is of great importance not only for policy makers (e.g. central banks, debt management offices) but also for investors. Given that prices of zero coupon bonds are not observed for many maturities, the term structure is not directly observed and needs to be estimated from coupon bond prices.

2 Data

Government securities included in the zero-coupon yield curve estimation need to be homogenous in terms of their risk characteristics and liquidity. For this reason, we use only fixed-coupon government bonds of all maturities, including zero-coupon bonds. Floating rate bonds, private placements, international bonds, and foreign currency bonds are excluded. We also exclude Treasury bills (Štátne pokladničné poukážky) and Railway bonds (Železnice Slovenskej Republiky). The reason for excluding Treasury bills is the fact that historical price data are scarce. Our data sample period starts in January 2003 and is determined by the availability of reliable bond price data. On average, we obtain 13 bonds per day. The zero-curve estimation does not take taxes into account as the marginal tax rate varies across investors.

Coupon bond prices are obtained mainly via Bloomberg and complemented with data from Reuters/Datastream. Bond prices obtained via Bloomberg are last traded prices on a given day. Prices from Reuters/Datastream are synthetic, i.e. aggregated from traded prices and model-based fair valuations. Around 88% of prices are from Bloomberg with the remainder coming from Reuters/Datastream. We perform a series of data quality checks and exclude outliers, mis-priced bonds, and bonds with the remaining time to maturity less than one month.

3 The model

Spot interest rate is the yield to maturity of a zero-coupon bond. Series of spot rates of different maturities defines the term structure. A coupon bond has a different spot rate for each cash flow. The term structure defined in terms of spot rates provides information about forward rates and vice versa. The relationship is as follows:

$$\left(1 + i_{t,m}\right)^{m} = \prod_{\tau=1}^{m}\left(1 + f_{t,\tau}\right) \qquad (1)$$

where $i_{t,m}$ is the spot interest rate for maturity m and $f_{t,\tau}$ denotes the forward rate observed at time t for period τ . Discount rate, denoted by $d_{t,m}$ represents the present value of one unit paid out at some future point m:

$$d_{t,m} = \frac{1}{\left(1+i_{t,m}\right)^{m}} \qquad (2)$$

Using the discount rates, we can write the price of a coupon bond as:

$$P_{j,t} + AI_{j,t} = \sum_{m=1}^{M_j} d_{t,m} \times C_j + d_{t,M_j} \times F_j \qquad (3)$$

where $P_{j,t}$ denotes the bond price that excludes accrued interest (clean price) and $AI_{j,t}$ represents accrued interest which is computed according to the corresponding day count convention that can vary across bonds. C_j is the coupon rate and F_j denotes the face value of a bond *j*.

We model yields and discount rates using the methodology proposed in Svensson (1994) which is an extension of the model outlined in Nelson and Siegel (1987).[1] The motivation for using this model is twofold. First, Nelson and Siegel (1987) method with less parameters does not provide enough flexibility, especially around the zero lower bound in the most recent years. Second, Svensson (1994) model is used to estimate the German yield curve (Schich, 1997) and the Euro area yield curve.[2] On a more general level, the main motivation for the functional form (as opposed to spline-based methods) is the fact that it can parsimoniously replicate most of the observed shapes of the yield curve. Spline-based methods are more appropriate in cases with a large number of securities available, which is not the case for Slovak Government bonds.

The Svensson model assumes the following functional relationship between the spot interest rate *i* and the time to maturity *m*:

$$i(m;b) = \beta_0 + \beta_1 \frac{1-\exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} + \beta_2 \left(\frac{1-exp\left(-\frac{m}{\tau_1}\right)}{\frac{m}{\tau_1}} - exp\left(-\frac{m}{\tau_1}\right) \right) + \beta_3 \left(\frac{1-exp\left(-\frac{m}{\tau_2}\right)}{\frac{m}{\tau_2}} - exp\left(-\frac{m}{\tau_2}\right) \right), \qquad (4)$$

where *b* is a vector of parameters $b = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ to be estimated. The corresponding forward rate can be written as:

$$f(m;b) = \beta_0 + \beta_1 exp\left(-\frac{m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} exp\left(-\frac{m}{\tau_1}\right) + \beta_3 \frac{m}{\tau_2} exp\left(-\frac{m}{\tau_2}\right) \qquad (5)$$

This functional form implies that $f \rightarrow \beta_0$ as $m \rightarrow \infty$ and $f \rightarrow \beta_0 + \beta_1$ as $m \rightarrow 0$, i.e. β_0 represents the forward rate at infinitely long horizon and $\beta_0 + \beta_1$ represents the forward rate at zero maturity. The Svensson model allows for two humps whose positions are controlled by τ_1 and τ_2, respectively. The direction and magnitude of the humps is controlled by β_2 and β_3 which explain more than 95 per cent of variation in the model. Clearly, β_0, τ_1, τ_2 all need to be positive. The corresponding discount function is defined as:

$$d(m;b) = exp\left(-\frac{i(m;b)}{100} m\right) \qquad (6)$$

[1] The extension by Svensson (1994) increases the flexibility of the Nelson-Siegel functional form by adding two parameters β_3 and τ_2 which allow for an additional hump.
[2] See https://www.ecb.europa.eu/stats/money/yc/html/index.en.html for more details.

4 The estimation

The functional form for the relationship between the spot rates and the time to maturity *m* given by Eq (4) captures main factors that drive observable yields to maturity for coupon bonds. In addition to these, there are other factors that are mostly bond-specific such as demand-supply considerations, tax-related effects, etc. These largely bond-specific effects are reflected in the pricing error. To obtain the zero coupon curve, we need to estimate parameters in vector *b* every trading day.

Every trading day, bonds are decomposed into cash flows and the respective timing of these cash flows. Using the discount function defined in Eq (6), we obtain a model price for a bond *j* as a function of a parameter vector *b* denoted by $P_j^{mod}(b_t)$:

$$P_j^{mod}(b_t) + AI_{j,t} = \sum_{m=1}^{M_j} d(m; b_t) \times C_j + d(M_j; b_t) \times F_j \quad (7)$$

Pricing error of a bond *j* is defined as:

$$\varepsilon_{t,j} = P_{t,j} - P_j^{mod}(b_t) \quad (8)$$

We minimize the squared price errors to estimate b_t:

$$min_{b_t} \sum_{j=1}^{n_t} (\varepsilon_{t,j}(b_t) \times w_{t,j})^2 \quad (9)$$

where $w_{t,j}$ denotes the weight for a bond *j* at time *t*. We overweight errors on short-term bonds in the estimation to ensure that the yield errors at the short-end of the yield curve are not excessive. We also regularize the estimation with three-month BRIBOR/EURIBOR rates converted to a zero-coupon bond equivalent. This is to anchor the short-end of the curve where not many short-term bonds are available. Given that the optimization problem given by Eq (9) is known to feature multiple local minima, see e.g. Gilli, Grosse, and Schumann (2010), we estimate b_t with a differential evolution algorithm instead of a direct or gradient-based search when needed. Once we have obtained estimates for b_t, zero-coupon interest rates and the corresponding discount rates are determined for a given maturity *m* by inserting the estimates $\widehat{b_t}$ into Eq (4) and Eq (6), respectively.

5 Measuring bond liquidity

Estimation of the term structure lends itself to tracking the aggregate level of liquidity of government bonds. The level of liquidity is related to availability of arbitrage capital. In case of government bonds, arbitrageurs exploit small mis-pricings in related government securities (e.g. similar maturity) and thus tie yields on these bonds closely together. Large price deviations are eliminated by arbitrageurs during normal times. Based on this premise, one can construct a simple measure of illiquidity by averaging the pricing errors for each trading day as proposed in Hu, Pan, and Wang (2013):

$$\text{illiq}_t = \sqrt{\frac{1}{N_t}\sum_{j=1}^{N_t}(y_t^j - y^j(b_t))^2}$$

where y_j is the observed coupon bond yield, $y^j(b_t)$ is its modeled counterpart and N_t is the number of coupon bonds at time t. To obtain $illiq_t$, we consider bonds with the remaining time to maturity between one and ten years. Even though there are bonds with maturities higher than ten year in the recent years, we exclude these to ensure the comparability of $illiq_t$ with the earlier period where all bonds have maturities less than ten years.

6 Available historical zero-coupon data

We publish the zero-coupon data starting at the maturity of one year. This is because we do not include Treasury bills in the estimation and there are usually not many liquid bonds with maturity below one year that would deliver a sufficient precision in the estimation at the short-end. Maximum maturity of the zero-coupon yield data is determined by the longest time to maturity available. Until May 2006, we publish data up to the maturity of ten years and up to 15 years thereafter. The data can be downloaded from the website of the Ministry of Finance of the Slovak Republic[3] and will be updated regularly. Codes with the model in Matlab are available upon a request from the authors.

7 Comparison with the German yield curve

To construct the zero-coupon spreads between the Slovak and the German term structure, we reconstruct the German yield curve using parameter estimates published on the Bundesbank website.[4]

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[3] Dataset is available at the following link:
http://www.finance.gov.sk/en/Default.aspx?CatID=713
[4] See:
https://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/Macro_economic_time_series/its_list_node.html?listId=www_s140_it03a .

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